TRINITY COLLEGE

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words _____

_____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	92	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section Two: Calculator-assumed   **65% (92 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8   **(6 marks)**

A classroom has a barrel containing a large number of blue crayons and yellow crayons.

(a)   14 crayons are randomly drawn from the barrel and placed, in the order, on a table. In terms of their colours, how many different orders are possible?   (1 mark)

(b)   An empty box is to be filled with some of the crayons. What is the smallest number of crayons that should be placed in the box to guarantee that it contains at least 7 blue or at least 9 yellow crayons? Justify your answer.   (2 marks)

(c)   Each of the 21 students in the classroom choose 5 crayons from the barrel. Use the pigeonhole principle to prove that at least 4 of the students will choose the same colour combination of crayons.   (3 marks)

Question 9   **(6 marks)**

Four figure numbers are to be formed from the digits $1, 2, 3, 4, 5$.

(a)   How many different four figure numbers can be formed

      (i)   if repetition of digits is allowed?   (1 mark)

      (ii)   without repetition of digits?   (1 mark)

(b)   How many of the numbers without repetition are greater than 4312?   (3 marks)

(c)   How many of the numbers without repetition are less than 4312?   (1 mark)

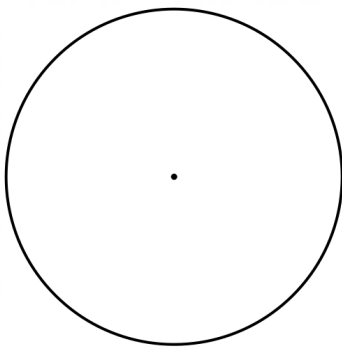**Question 10**   **(9 marks)**

(a)   Points P, Q, R and S lie in order on the circumference of the circle with centre O so that
        $PQ = 8.8$ cm, $PS = 10.5$ cm, and PR and QS are diameters. Determine, with brief reasons
        and to the nearest degree, the sizes of $\angle PQS$, $\angle PRS$, $\angle POS$ and $\angle RPS$.   (5 marks)
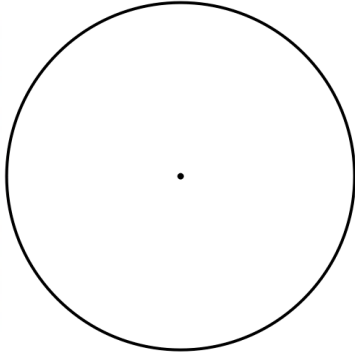
(b)   Points A, B and C lie on the circumference of a circle of radius 26 cm, so that $BC = 28$ cm
        and $AC = 45$ cm. Prove by contradiction that the midpoint of chord AB is not the centre of
        the circle.   (4 marks)

See next page

Question 11   **(8 marks)**

Relative to boat O at anchor in a lake, four buoys A, B, C and D have the following position vectors (with distances in metres):

$$\overrightarrow{OA} = (210, -935), \qquad \overrightarrow{OB} = (90, -200), \qquad \overrightarrow{OC} = (330, 360), \qquad \overrightarrow{OD} = (390, -515).$$

(a)   Prove that the quadrilateral with vertices $ABCD$ is a trapezium, but not a parallelogram.
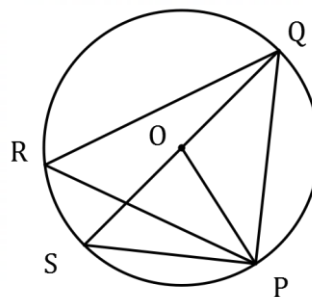
(5 marks)

(b)   Boat X motors directly from D to B with a constant velocity in 2 minutes and 30 seconds. Determine the velocity in component form, and hence the speed, of boat X.   (3 marks)

Question 12 (8 marks)
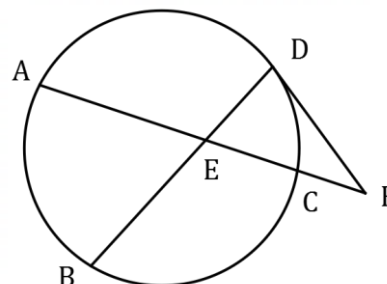
(a) In the diagram (not to scale), points P, Q, R and S lie on a circle with centre O and diameter QS.

If $\angle QRP = 59°$, determine the size of

(i) $\angle QOP$. (1 mark)

(ii) $\angle QSP$. (1 mark)

(iii) $\angle SQP$. (1 mark)

(iv) $\angle POS$. (1 mark)

(b) In the diagram (not to scale), FD is a tangent to the circle at D. Secant AF cuts chord BD at E, and the circle at C.

$FC = 4$ cm, $CA = 21$ cm, $DF = EF$, and BE is 2 cm longer than DF.

Determine the length of DE. (4 marks)

See next page

Question 13 **(7 marks)**

(a) Consider the letters in the word DEMATERIALISE. Determine the number of different

   (i) combinations of 3 letters chosen from the consonants in the word. (1 mark)

   (ii) permutations of all the letters in the word. (2 marks)

(b) Four-digit pin codes such as 3812 are made by randomly choosing four different digits from those in the number 12 345 678. Determine the fraction of all such possible pin codes that start with 12 or end in 8. (4 marks)

Question 14   **(8 marks)**

Two vectors are $\mathbf{a} = \begin{pmatrix} x \\ -5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 6 \\ y \end{pmatrix}$, where x and y are constants.

(a)   When $x = 11$ and $y = 7$, determine

     (i) $\mathbf{a} \cdot \mathbf{b}$.   (1 mark)

     (ii)   a unit vector in the same direction as $\mathbf{a} - \mathbf{b}$, in exact form.   (2 marks)

     (iii)   the angle between the directions of the unit vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, rounded to one
           decimal place.   (2 marks)

(b)   Determine the value of x and the value of y when $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, $x < y$, and
     $|\mathbf{a} + \mathbf{b}| = 12.2$.   (3 marks)

See next page

Question 15   **(8 marks)**

(a)   The vertices of triangle PQR lie on a circle with centre O so that QR is a diameter. The midpoint of PQ is M. Prove that OM is parallel to RP.   (4 marks)

(b)   The vertices of a kite lie on the circumference of a circle. Each longer side of the kite is four times the length of the adjacent shorter side. If the area of the kite is 64 cm², determine the radius of the circle.   (4 marks)

Question 16 (9 marks)

The diagram at right, not to scale, shows forces F_1 and F_2 acting in the same vertical plane on a small hook fixed to a vertical wall. F_1 has magnitude 147 N and acts at an angle of elevation of 22° and F_2 has magnitude 195 N and acts at an angle of depression of 42°.
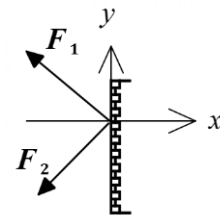
The resultant of F_1 and F_2 is R.

(a) Sketch a triangle to show the relationship between F_1, F_2 and R. (1 mark)

(b) Determine, with reasoning, the magnitude of R and the acute angle it makes with the wall. (5 marks)

(c) The wall exerts a force on the hook of equal magnitude to R but in the opposite direction. Express this force using unit vectors i and j. (3 marks)

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Question 17 **(9 marks)**

A small boat has a cruising speed of 17 km/h in still water. The boat leaves point A at 8:30 am and travels to point B, 7.7 km due east of A, where it turns and travels to point C, 3.6 km due north of B. The boat then returns to A. A current of 2.6 km/h runs in an easterly direction throughout the area.

(a) Determine the time taken to travel from point A to point B. (1 mark)

(b) Determine the time taken to travel from point B to point C. (2 marks)

(c)   Determine the time taken for the return leg, from point C to point A.   (5 marks)

(d)   What time does the boat return to A?   (1 mark)

See next page

Question 18   **(6 marks)**

Consider the identity $^nC_r = {}^{(n+1)}C_r - {}^nC_{(r-1)}$.

(a)   With $n = 4$ and $r = 3$, show that the left and right sides of the identity are equal.   (1 mark)

(b)   State all necessary restrictions on n and r for the identity to exist and to be valid.

(2marks)

(c)   Prove the identity is always true, subject to all necessary restrictions.   (3 marks)
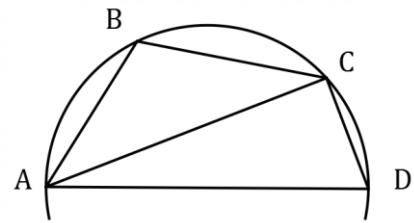
Question 19 (8 marks)

(a) Points A, B, C and D lie on an arc of a circle
as shown, so that AD is a diameter.

Let $\alpha = \angle CAD$ and $\beta = \angle ABC$.

Determine in simplest form the relationship between α and β. (3 marks)

(b) Tangents from X touch a circle at P and Q. Diameter PR and tangent XQ are both
extended to meet at S. Prove that $\angle PXQ = 2\angle RQS$. (5 marks)

End of questions

Supplementary page

Question number: _____